Conversion between Cylindrical and Cartesian Coordinates

It can be shown that the rectangular coordinates and cylindrical coordinates in Fig.1 are related as follows:

$x = r\cos\theta$ $y = r\sin\theta$ $z = z$ $r = \sqrt{x^2 + y^2}$ $\tan\theta = \frac{y}{x}$ $z = z$ $0 \le \theta < 2\pi$

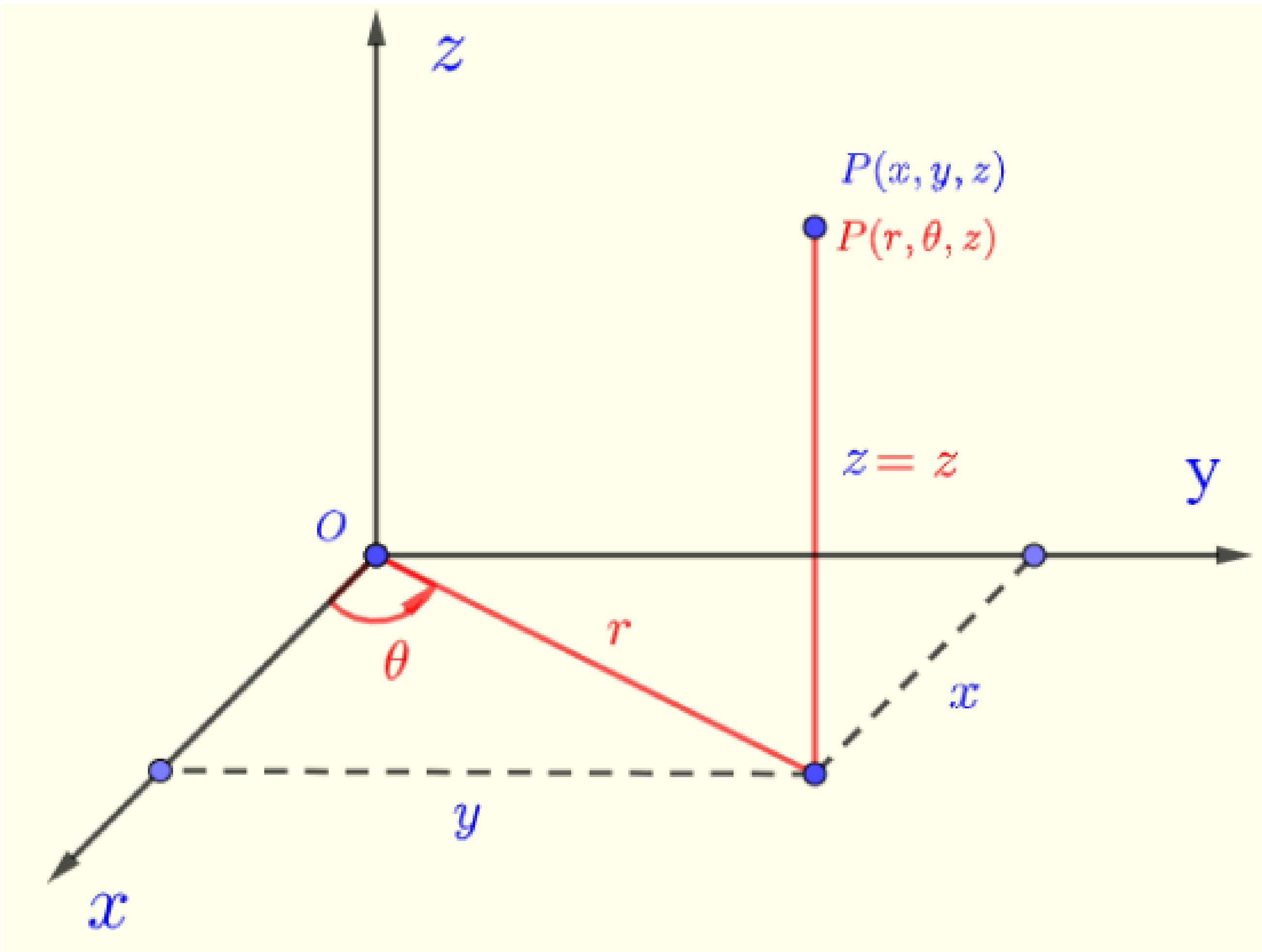

Fig.1 - Rectangular and cylindrical coordinates

The Cartesian coordinate system provides a straightforward way to describe the location of points in space. Some surfaces, however, can be difficult to model with equations based on the Cartesian system. This is a familiar problem; recall that in two dimensions, polar coordinates often provide a useful alternative system for describing the location of a point in the plane, particularly in cases involving circles. In this section, we look at two different ways of describing the location of points in space, both of them based on extensions of polar coordinates. As the name suggests, cylindrical coordinates are useful for dealing with problems involving cylinders, such as calculating the volume of a round water tank or the amount of oil flowing through a pipe. Similarly, spherical coordinates are useful for dealing with problems involving spheres, such as finding the volume of domed structures.

Cylindrical Coordinates

When we expanded the traditional Cartesian coordinate system from two dimensions to three, we simply added a new axis to model the third dimension. Starting with polar coordinates, we can follow this same process to create a new three-dimensional coordinate system, called the

cylindrical coordinate system

. In this way, cylindrical coordinates provide a natural extension of polar coordinates to three dimensions.

Definition: The

Cylindrical Coordinate System

In the

cylindrical coordinate system

, a point in space (Figure 1) is represented by the ordered triple (r,θ,z)(r,θ,z), where

- (r,θ)(r,θ) are the polar coordinates of the point's projection in the xyxy-plane
- zz is the usual zz-**coordinate** in the Cartesian coordinate system

In the xyxy-plane, the right triangle shown in Figure 1 provides the key to Transformation between cylindrical and Cartesian, or rectangular, coordinates.

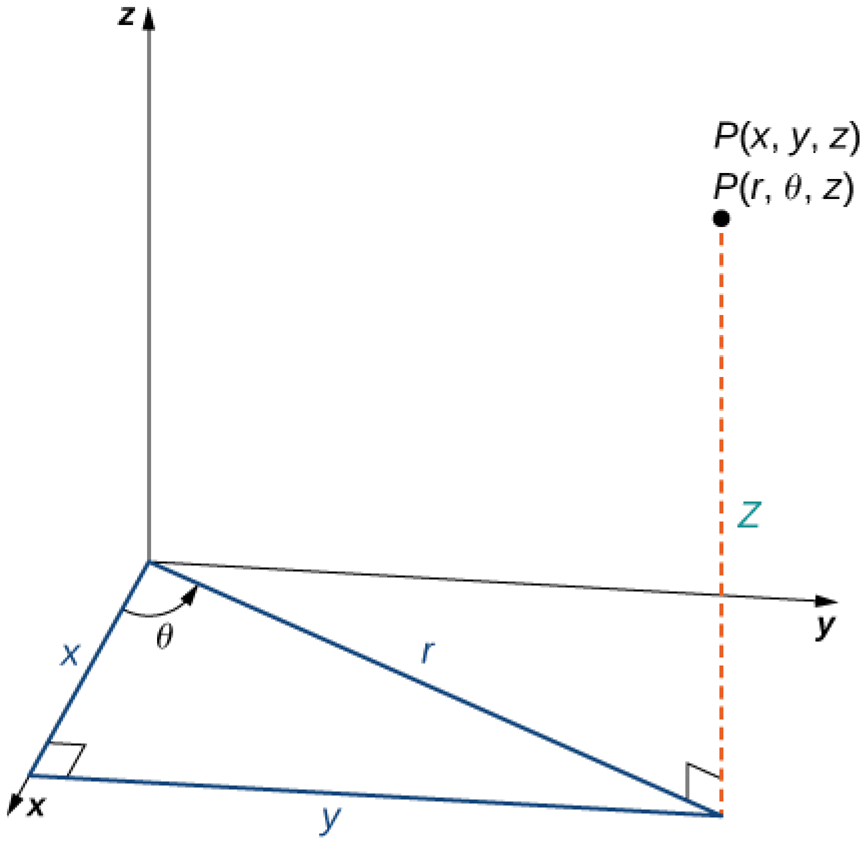

Figure 1: The right triangle lies in the xyxy-plane. The length of the hypotenuse is rr and θθ is the measure of the angle formed by the positive xx-axis and the hypotenuse. The zz-coordinate describes the location of the point above or below the xyxy-plane.